1) La función $\frac{3x}{5x-21}$ es homográfica con asíntota vertical en x=21/5. Tiende a $-\infty$ por izquierda y a $+\infty$ por derecha. Por lo tanto, tanto el supremo como el ínfimo de A deben corresponder a los n más cercanos a 21/5: 4 y 5.

Si n=4, $\frac{3n}{5n-21}=-12$ (mínimo). Es menor a todos los elementos anteriores. Para n>5, la sucesión es decreciente, pero su límite es 0.

Si n=5, $\frac{3n}{5n-21}=\frac{15}{4}$ (máximo). Es mayor a todos los elementos anteriores. Para n>5, la sucesión es decreciente.

2)

$$\lim_{(x,y)\to(0,-1)} \frac{(y+1)sen^2\left(x^2-2(y+1)^2\right)}{x^4-4(y+1)^4}$$

$$\lim_{(x,y)\to(0,0)} \frac{(e^y-1)^2}{x-y}=0$$

$$\lim_{(x,-1)\to(0,-1)} \frac{(y+1)sen^2\left(x^2-2(y+1)^2\right)}{x^4-4(y+1)^4}=0$$

$$\lim_{(0,y)\to(0,-1)} \frac{sen^2\left(-2(y+1)^2\right)}{-4(y+1)^3}=\lim_{(0,y)\to(0,-1)} \frac{sen\left(-2(y+1)^2\right)}{-2(y+1)^2}\frac{sen\left(-2(y+1)^2\right)}{2(y+1)}=$$

$$\lim_{(0,y)\to(0,-1)} \frac{sen\left(-2(y+1)^2\right)}{2(y+1)}\frac{-(y+1)}{-(y+1)}=\lim_{(0,y)\to(0,-1)} \frac{sen\left(-2(y+1)^2\right)}{-2(y+1)^2}\left(-(y+1)\right)=$$

$$\lim_{(0,y)\to(0,-1)} \left(-(y+1)\right)=0$$

Si hay límite, es 0

$$\lim_{(x,y)\to(0,-1)} \frac{(y+1)\cdot sen^2\left(x^2-2(y+1)^2\right)}{x^4-4(y+1)^4}=\lim_{(x,y)\to(0,-1)} \frac{(y+1)\cdot sen^2\left(x^2-2(y+1)^2\right)}{\left(x^2+2(y+1)^2\right)\left(x^2-2(y+1)^2\right)}$$

$$=\lim_{(x,y)\to(0,-1)} \frac{(y+1)\cdot sen\left(x^2-2(y+1)^2\right)}{\left(x^2+2(y+1)^2\right)}\frac{sen\left(x^2-2(y+1)^2\right)}{\left(x^2-2(y+1)^2\right)}$$

$$= \lim_{(x,y)\to(0,-1)} \frac{(y+1)\cdot sen\left(x^2-2(y+1)^2\right)}{\left(x^2+2(y+1)^2\right)}\cdot 1 =$$

$$= \lim_{(x,y)\to(0,-1)} \frac{(y+1)\cdot\left(x^2-2(y+1)^2\right)}{\left(x^2+2(y+1)^2\right)}\cdot\frac{sen\left(x^2-2(y+1)^2\right)}{\left(x^2-2(y+1)^2\right)} =$$

$$= \lim_{(x,y)\to(0,-1)} \frac{(y+1)\cdot\left(x^2-2(y+1)^2\right)}{\left(x^2+2(y+1)^2\right)}\cdot 1$$

$$= \lim_{(x,y)\to(0,-1)} \frac{(y+1)\cdot\left(x^2-2(y+1)^2\right)}{\left(x^2+2(y+1)^2\right)} < \lim_{(x,y)\to(0,-1)} \frac{(y+1)\cdot\left(x^2-2(y+1)^2\right)}{\left(2x^2+2(y+1)^2\right)}$$

$$< \lim_{(x,y)\to(0,-1)} \frac{\|x,(y+1)\|\cdot\left(x^2-2(y+1)^2\right)}{2\|x,(y+1)\|^2} = \lim_{(x,y)\to(0,-1)} \frac{x^2-2(y+1)^2}{2\|x,(y+1)\|} <$$

$$\lim_{(x,y)\to(0,-1)} \frac{\left(x^2-2(y+1)^2\right)+3(y+1)^2}{2\|x,(y+1)\|} = \lim_{(x,y)\to(0,-1)} \frac{x^2+(y+1)^2}{2\|x,(y+1)\|} =$$

$$\lim_{(x,y)\to(0,-1)} \frac{\|x,(y+1)\|^2}{2\|x,(y+1)\|} = \lim_{(x,y)\to(0,-1)} \frac{1}{2}\|x,(y+1)\|$$

$$\left|\frac{(y+1)\cdot\left(x^2-2(y+1)^2\right)}{\left(x^2+2(y+1)^2\right)}\right| < \frac{1}{2}\|x,(y+1)\| < \frac{1}{2}\delta$$

$$\delta = 2\varepsilon$$

$$\left|\frac{k\cdot\left(x^2-2k^2\right)}{\left(x^2+2k^2\right)} - 0\right| < \varepsilon$$

$$\lim_{(x,0)\to(0,0)} \frac{0}{x} = 0$$

$$\lim_{(0,y)\to(0,0)} \frac{\left(e^y-1\right)^2}{-y} = \frac{2\left(e^y-1\right)e^y}{-1} = -2\left(e^y-1\right)e^y = 0$$

$$\lim_{(x,x)\to(0,0)} \frac{\left(e^x-1\right)^2}{x-x} = \lim_{(x,x)\to(0,0)} \frac{-2\left(e^{2x}-e^x\right)}{0} = \infty \Rightarrow \nexists \lim$$

3)

$$\frac{\delta f}{\delta x}(0,0) = \lim_{h\to 0}\frac{f(h,0)}{h} = \lim_{h\to 0}\frac{0}{h^7} = 0$$

$$\frac{\delta f}{\delta y}(0,0) \lim_{h\to 0}\frac{f(0,h)}{h} = \lim_{h\to 0}\frac{0}{\left(h^2+h^3\right)^2} = 0$$

$$\lim_{(xy)\to(0,0)} \frac{sen^2(xy)\left(e^{y^2+(x^2+y^2)^{\frac{3}{2}}}-1\right)}{\|(xy)\|\left(y^2+(x^2+y^2)^{\frac{3}{2}}\right)^2}=0$$

$$\lim_{(xy)\to(0,0)} \frac{sen^2(xy)\left(e^{y^2+\|(xy)\|^{2\frac{3}{2}}}-1\right)}{\|(xy)\|\left(y^2+\left(\|(xy)\|^2\right)^{\frac{3}{2}}\right)^2}=\frac{sen^2(xy)\left(e^{y^2+\|(xy)\|^3}-1\right)}{\|(xy)\|\left(y^2+\|(xy)\|^3\right)^2}$$

$$<\lim_{(xy)\to(0,0)} \frac{sen^2(xy)\left(e^{y^2+\|(xy)\|^3}-1\right)(xy)^2}{\left(y^2+\|(xy)\|^3\right)^2(xy)^2}=\lim_{(xy)\to(0,0)} \frac{\left(e^{y^2+\|(xy)\|^3}-1\right)(xy)^2}{\left(y^2+\|(xy)\|^3\right)^2}$$

$$\lim_{(xy)\to(0,0)} \frac{\left(e^{y^2+\|(xy)\|^3}-1\right)(xy)^2}{\left(y^2+\|(xy)\|^3\right)\left(y^2+\|(xy)\|^3\right)}=$$

$$\left(\begin{array}{l} y^2+\|(xy)\|^3=t \\ \lim_{t\to 0}\frac{e^t-1}{t}\underset{L'H}{=}\lim_{t\to 0}\frac{e^t}{1}=1 \end{array}\right)$$

$$=\lim_{(xy)\to(0,0)} \frac{(xy)^2}{\left(y^2+\|(xy)\|^3\right)}<\lim_{(xy)\to(0,0)} \frac{\|(xy)\|^4}{\|(xy)\|^3}=$$

$$\lim_{(xy)\to(0,0)}\|(xy)\|=0$$

Es diferenciable

4)

$$z=f(2,-1)+\frac{\delta f}{\delta x}(2,-1)(x-2)+\frac{\delta f}{\delta y}(2,-1)(y+1)$$

$$\frac{\delta f}{\delta x}(-2,1)=-4$$

$$f(2,-1)=-1$$

$$D\left(\frac{\delta f}{\delta y}\circ g\right)(2,-1)=D\frac{\delta f}{\delta y}\left(g(2,-1)\right)\cdot Dg(2,-1)$$

$$(-20,0)=D\frac{\delta f}{\delta y}\left(2,\,f(2,-1)\right)\cdot Dg(2,-1)$$

$$(-20,0) = D\frac{\delta f}{\delta y}(2,-1)\cdot Dg(2,-1)$$

$$g_1(x,y) = 7-5y^2$$

$$g_2(x,y) = f(xy)$$

$$Dg_{1x} = 0$$

$$Dg_{1y} = -10y$$

$$Dg_{1y}(2,-1) = 10$$

$$Dg_{2x} = \frac{\delta f}{\delta x}$$

$$Dg_{2x}(2,-1) = \frac{\delta f}{\delta x}(2,-1) = -4$$

$$Dg_{2y}(2,-1) = \frac{\delta f}{\delta y}(2,-1)$$

$$Dg(2,-1) = \begin{pmatrix} 0 & 10 \\ -4 & \frac{\delta f}{\delta y}(2,-1) \end{pmatrix}$$

$$\begin{pmatrix} \frac{\delta^2 f}{\delta y \delta x}(2,-1) & \frac{\delta^2 f}{\delta y^2}(2,-1) \end{pmatrix}\cdot\begin{pmatrix} 0 & 10 \\ -4 & \frac{\delta f}{\delta y}(2,-1) \end{pmatrix} = (-20,0)$$

$$\frac{\delta^2 f}{\delta y \delta x}(2,-1)\cdot 0 - 4\frac{\delta^2 f}{\delta y^2}(2,-1) = -20$$

$$\frac{\delta^2 f}{\delta y^2}(2,-1) = -5$$

La ecuación del plano tangente al gráfico de $\frac{\delta f}{\delta x}$ en el punto $(-2,1,-4)$ es $z = 4x + y + 3$.

$$z = \frac{\delta^2 f}{\delta x^2}(x+2) + \frac{\delta^2 f}{\delta x \delta y}(y-1) + \frac{\delta f}{\delta x}(-2,1)$$

$$z = \frac{\delta^2 f}{\delta x^2}(-2,1)x + 2\frac{\delta^2 f}{\delta x^2}(-2,1) + \frac{\delta^2 f}{\delta x \delta y}(-2,1)y - \frac{\delta^2 f}{\delta x \delta y}(-2,1) - 4$$

$$\frac{\delta^2 f}{\delta x^2}(-2,1) = 4$$

$$\frac{\delta^2 f}{\delta x \delta y}(-2,1) = 1$$

Por la matriz

$$10\frac{\delta^2 f}{\delta y \delta x}(2,-1)+\frac{\delta^2 f}{\delta y^2}(2,-1)\cdot\frac{\delta f}{\delta y}(2,-1)=0$$

$$10-5\cdot\frac{\delta f}{\delta y}(2,-1)=0$$

$$\frac{\delta f}{\delta y}(2,-1)=2$$

Ecuación del plano tangente a f

$$z=-1-4(x-2)+2(y+1)$$

$$z=-4x+2y+9$$