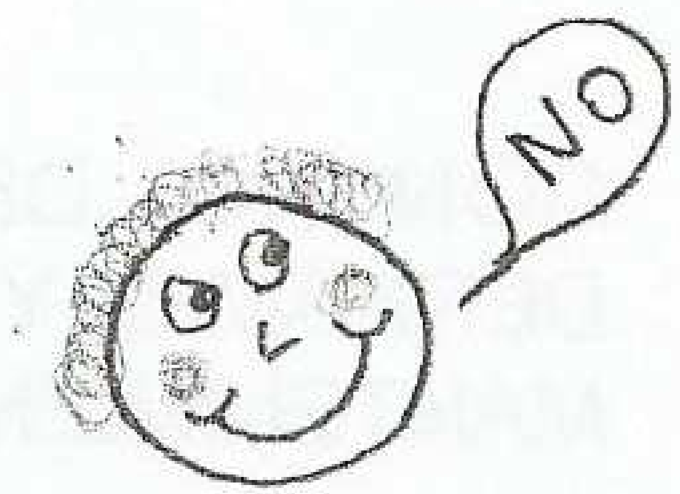
FINAL 12/10/11



1) Sea $f:\mathbb{R}^2 \to \mathbb{R}$, C^1. Probar que f es diferenciable.

2) Hallar el punto de la superficie $S = \{(x,y,z) \in \mathbb{R}^3 \,/\, z^2 = (x-1)^2 + (y-1)^2\}$ más cercanos al $(0,0,0)$.

3) Sea $f:\mathbb{R}^2 \to \mathbb{R}$ continua, y $F(x,y) = \int_0^y \int_0^x f(s,t)\,ds\,dt$. Probar que $F(x,y)$ es diferenciable.

4) Hallar el volumen encerrado entre las superficies
$S_1 = \{(x,y,z) \in \mathbb{R}^3 \,/\, z = 5 - x^2 - y^2\}$ y $S_2 = \{(x,y,z) \in \mathbb{R}^3 \,/\, z = \sqrt{x^2+y^2}\}$.

12/10/11

② Hallar el punto de $S = \{(x,y,z) \in \mathbb{R}^3 : \overbrace{z^2 = (x-1)^2 + (y-1)^2}^{G}\}$ más cerca del ⓪

Quiero minimizar $\sqrt{x^2+y^2+z^2} \Rightarrow$ $x^2+y^2+z^2 = d$

Hipótesis de Lagrange g es C^1 y d es diferenciable

$\Rightarrow$ puedo aplicar LAGRANSH $\Rightarrow \nabla d = \lambda(\nabla g)$

$(2x, 2y, 2z) = \lambda.(2x-2, 2y-1, -2z)$

$2x = (2x-2)\lambda$

$2y = (2y-2)\lambda$

$2z = -2\lambda z \Rightarrow z \neq 0 \Rightarrow \boxed{\lambda = -1}$

$\Rightarrow 2x = -2x+2$ $\Rightarrow x = 1/2$ $\Rightarrow y = 1/2$ $z = 1/\sqrt{2}$ PUNTO MÁS CER-K-NO

$\boxed{z=0}$

$2x = 2x\lambda - 2\lambda$

$2x - 2x\lambda = -2\lambda$

$x(2-2\lambda) = -2\lambda$

$\boxed{x = \frac{-2\lambda}{2-2\lambda}}$

$\boxed{y = \frac{-2\lambda}{2-2\lambda}}$

$\lambda \neq 1$

$0 = \left(\frac{-2\lambda}{2-2\lambda} - 1\right)^2 + \cdots$

$2 = -2\lambda$

$2 - 2\lambda = -2\lambda$

$2 = 0$

Abs.

$\boxed{\lambda = 1}$

$2x = 2x - 2$

$0 = -2$

Abs

③ $f:\mathbb{R}^2 \to \mathbb{R}$ continua $\quad F(x,y) = \int_0^y \int_0^x f(s,t)\,ds\,dt$. Probar que $F(x,y)$ es dif

$$\cancel{\frac{\partial F}{\partial y} = \frac{\partial}{\partial x}\int_0^y\int_0^x f(s,t)\,ds\,dt = \int_0^x f(s,y)\,ds}$$

Quiero ver que $G(x) = \int_0^x f(s,t)\,ds$, es continua.

Como f es continua por TFCI $\Rightarrow G(x)$ es continua. Entonces $\int_0^x f(s,t)\,ds$ es continua $\Rightarrow \frac{\partial F}{\partial y} = \int_0^x f(s,y)\,ds$ (Por TFCI)

$\Rightarrow \nabla F = (F_x, F_y)$. Hay $n-1$ derivadas parciales continuas

$$\boxed{\Rightarrow \text{diferenciable}}$$

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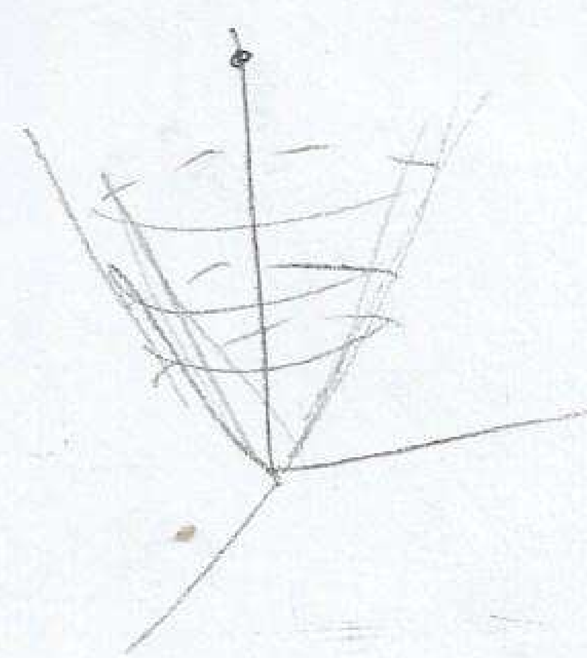
④ Hallar el volumen comprendido entre

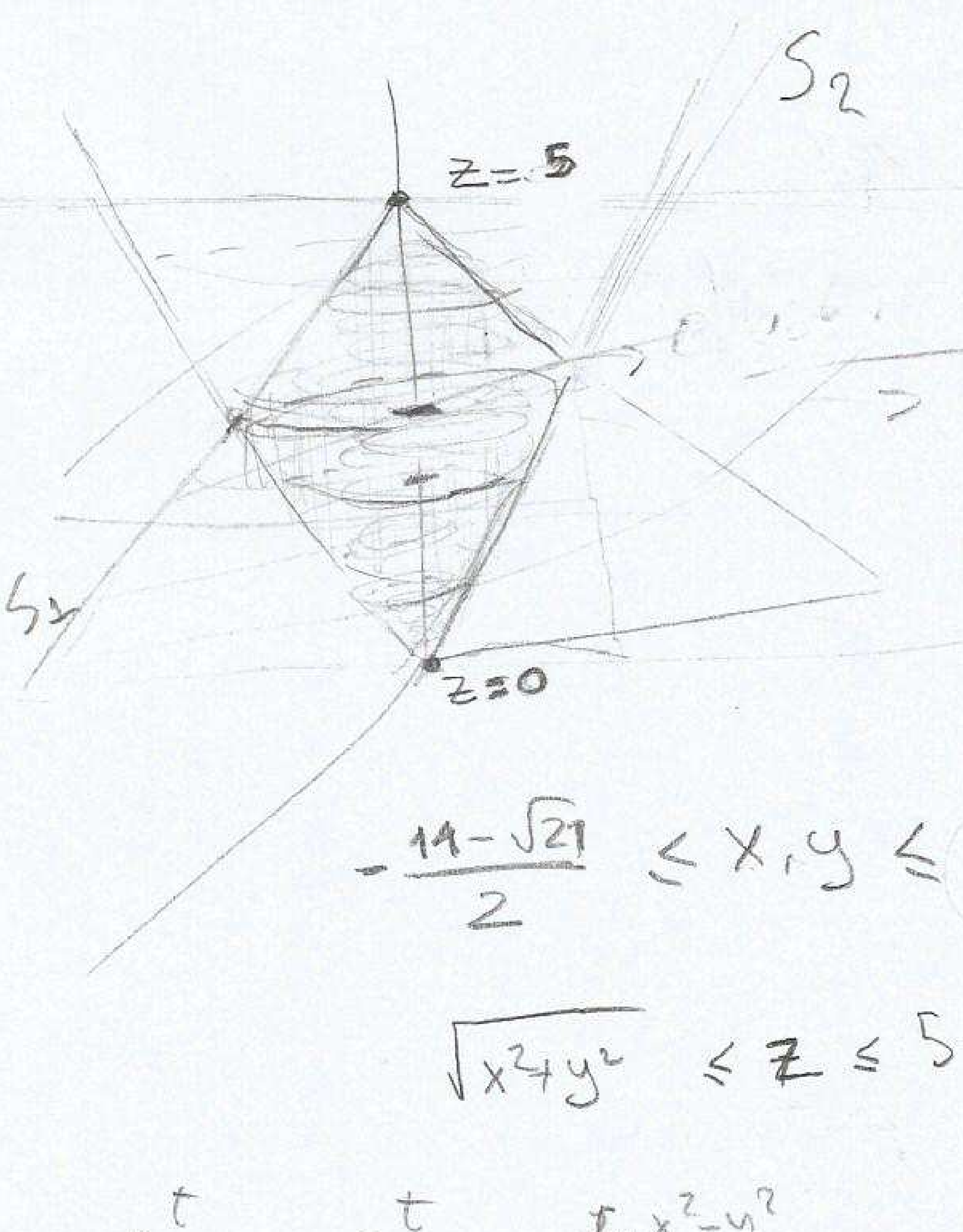
$S_1 = \{(x,y,z) \in \mathbb{R}^3 : z = 5 - x^2 - y^2\} \rightarrow 5 - z = x^2 + y^2$

$S_2 = \{(x,y,z) \in \mathbb{R}^3 : z = \sqrt{x^2+y^2}\} \rightarrow z^2 = x^2 + y^2$

$z^2 = x^2 + y^2$

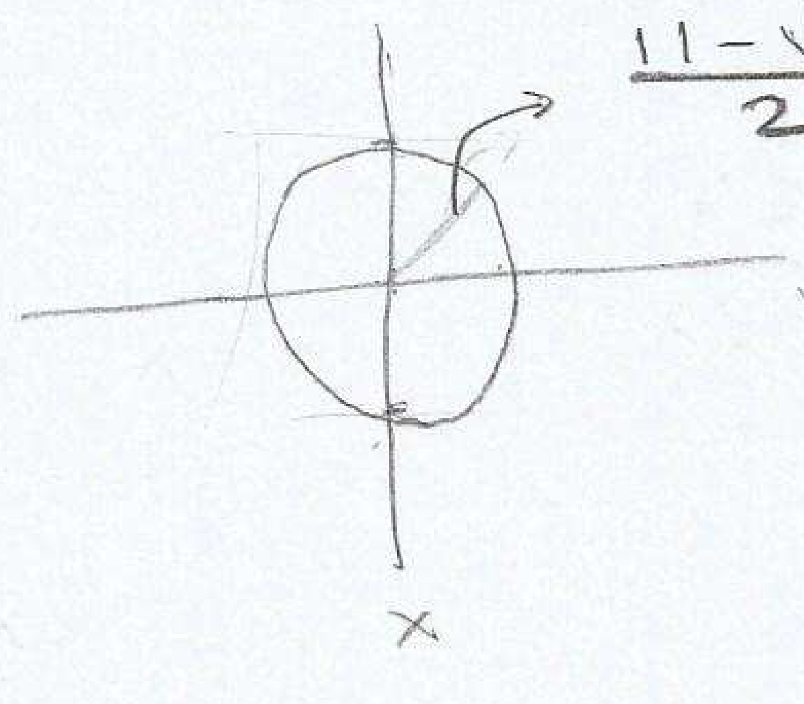


$S_1 = 5 - (x^2+y^2) = z \Rightarrow 5 - z = x^2 + y^2$



$$\left.\begin{array}{l} z^2 = x^2+y^2 \\ 5 - z = x^2+y^2 \end{array}\right\} \quad z^2 + z - 5 = 0 \qquad z = \frac{-1+\sqrt{21}}{2}$$

$\Rightarrow$ corte con plano $z = 2{,}5$



$$-\frac{11-\sqrt{21}}{2} \leq x, y \leq \frac{11-\sqrt{21}}{2}$$

$$\sqrt{x^2+y^2} \leq z \leq 5 - x^2 - y^2$$

$$\int_{-T}^{T}\int_{-T}^{T}\int_{\sqrt{x^2+y^2}}^{5-x^2-y^2} 1 \,dz\,dy\,dx = \int_{-T}^{T}\int_{-T}^{T} 5 - x^2 - y^2 - \sqrt{x^2+y^2}\,dy\,dx$$

$$= \int_{-T}^{T}\int_{-T}^{T} 5 - x^2 - y^2\,dy\,dx - \int_{-T}^{T}\int_{-T}^{T}\sqrt{x^2+y^2}\,dy\,dx =$$