1) a) A es una sucesión oscilante, pero la distancia de a_n al 0 se va reduciendo a medida que aumenta *n*. Luego, los primeros dos términos son el sup(A) e ínf (A)

$$ínf\,(A) = a_1 = \frac{-1}{3} = mín(A)$$

$$\sup(A) = a_2 = \frac{1}{5} = máx(A)$$

1) b) $b_n = \left|\frac{(-1)^n}{2n+1}\right| = \frac{1}{2n+1}$. Es una sucesión monótona y decreciente.

$$\sup(B) = b_1 = \frac{1}{5} = máx(B)$$

$$ínf\,(B) = \lim_{n\to+\infty} b_n = 0$$

$$\nexists mín(B)$$

2) $$\lim_{(x,y)\to(1,-1)} \frac{(y+1)g(x+y^2)}{\left(3(x-1)^2+2(y+1)^2\right)^{1/3}}$$

$$\lim_{(x,-1)\to(1,-1)} \frac{0}{\left(3(x-1)^2\right)^{1/3}} = 0$$

$$\lim_{(x,y)\to(1,-1)} \frac{(y+1)g(x+y^2)}{\left(3(x-1)^2+2(y+1)^2\right)^{1/3}} = \lim_{(x,y)\to(1,-1)} \frac{(x+y^2)^2(y+1)\frac{g(x+y^2)}{(x+y^2)^2}}{\left(3(x-1)^2+2(y+1)^2\right)^{1/3}}$$

$$= 3\cdot \lim_{(x,y)\to(1,-1)} \frac{(x+y^2)^2(y+1)}{\left(3(x-1)^2+2(y+1)^2\right)^{1/3}}$$

$$\le 3\cdot \lim_{(x,y)\to(1,-1)} (x+y^2)^2 \cdot \lim_{(x,y)\to(1,-1)} \frac{\|(x-1),(y+1)\|}{\left(2\|(x-1),(y+1)\|^2+2\|(x-1),(y+1)\|^2\right)^{1/3}}$$

$$= 3\cdot 4\cdot \lim_{(x,y)\to(1,-1)} \frac{\|(x-1),(y+1)\|}{\left(4\|(x-1),(y+1)\|^2\right)^{1/3}} = 12\cdot \lim_{(x,y)\to(1,-1)} \frac{\|(x-1),(y+1)\|}{\sqrt[3]{4}\|(x-1),(y+1)\|^{2/3}}$$

.

$$= \frac{12}{\sqrt[3]{4}}\cdot \lim_{(x,y)\to(1,-1)} \|(x-1),(y+1)\|^{1/3} = \frac{12}{\sqrt[3]{4}}\cdot 0 = 0$$

3) a)

$$\lim_{h\to 0}\frac{f\left((x_0 y_0)+h(v_1,v_2)\right)-f(0,0)}{h\|v_1,v_2\|}$$

$$\|v_1,v_2\|=1$$

$$\lim_{h\to 0}\frac{f(hv_1,hv_2)}{h}$$

$$\lim_{h\to 0}\frac{hv_1|hv_2|\left(h^2 v_1{}^2+h^2 v_2{}^2\right)}{\left(|h|^3|v_1|^3+h^6 v_2{}^6\right)h}$$

$$\lim_{h\to 0}\frac{v_1|hv_2|\left(v_1{}^2+v_2{}^2\right)h^3}{\left(v_1{}^3+|h|^3 v_2{}^6\right)|h|^3 h}$$

$$\lim_{h\to 0}\frac{v_1|v_2|}{v_1{}^3+|h|^3 v_2{}^6}$$

$$\frac{v_1|v_2|}{v_1{}^3}$$

$\exists$ si $v_1\neq 0$

si $(v_1,v_2)=(0,1)$

$$\frac{\delta f}{\delta v}=\lim_{h\to 0}\frac{f(0,h)}{h}=\frac{0}{h^7}=0\Rightarrow\exists$$

3) b)

$$f(x,y)=\begin{cases}\dfrac{x|y|\left(x^2+y^2\right)}{|x|^3+y^6} & (x,y)\neq(0,0)\\ 0 & (x,y)=(0,0)\end{cases}$$

$$\frac{\delta f}{\delta x}=\lim_{h\to 0}\frac{f(h,0)}{h}=\frac{0}{h^4}=0$$

$$\frac{\delta f}{\delta y}=\lim_{h\to 0}\frac{f(0,h)}{h}=\frac{0}{h^7}=0$$

$$\lim_{xy\to(0,0)}\left|\frac{\dfrac{x|y|\left(x^2+y^2\right)}{|x|^3+y^6}}{\|(xy)\|}\right|\overset{?}{=}0$$

$$\lim_{xy\to(0,0)}\frac{|x||y|\|(xy)\|^2}{\left||x|^3+y^6\right|\|(xy)\|}=\lim_{xy\to(0,0)}\frac{|x||y|\|(xy)\|}{\left||x|^3+y^6\right|}<\lim_{xy\to(0,0)}\frac{\|(xy)\|^3}{\left||x|^3+y^6\right|}$$

$$\underset{(x,x)\to(0,0)}{lím}\frac{\frac{x|x|\left(x^2+x^2\right)}{|x|^3+x^6}}{\|(x,x)\|}=\underset{(x,x)\to(0,0)}{lím}\frac{x|x|\left(2x^2\right)}{\sqrt{2}|x|\left(|x|^3+x^6\right)}=\underset{(x,x)\to(0,0)}{lím}\frac{x\left(2x^2\right)}{\sqrt{2}\left(|x|^3+x^6\right)}$$

$$=\frac{2}{\sqrt{2}}\underset{(x,x)\to(0,0)}{lím}\frac{x^3}{|x|^3+x^6}=\frac{2}{\sqrt{2}}\underset{(x,x)\to(0,0)^+}{lím}\frac{1}{1+x^3}=\frac{2}{\sqrt{2}}$$

$$\underset{(x,0)\to(0,0)}{lím}\frac{\frac{x|0|\left(x^2+0^2\right)}{|x|^3+0^6}}{\|(0x)\|}=\frac{0}{x^4}=0\Rightarrow\nexists lím$$

4) $g_1(xy)=\ln(xy+1)+y\cos(\pi x)$

$$\frac{\delta g_1}{\delta x}(xy)=\frac{y}{xy+1}-\pi y\sin(\pi x)$$

$$\frac{\delta g_1}{\delta y}(xy)=\frac{x}{xy+1}+\cos(\pi x)$$

$$g_2(xy)=e^{3x}+4y$$

$$\frac{\delta g_2}{\delta x}(xy)=3e^{3x}$$

$$\frac{\delta g_2}{\delta y}(xy)=4$$

$$g(0,0)=(0,1)$$

$$z=3+2x+3y\Rightarrow$$

$$\frac{\delta f\circ g}{\delta x}(0,0)=2$$

$$\frac{\delta f\circ g}{\delta y}(0,0)=3$$

ojo que esto es solo en el (0,0)

$$f\circ g(0,0)=3$$

$$D(f\circ g)(0,0)=Df\,(g(0,0))\cdot Dg(0,0)$$

$$Df\,(x,y)=\begin{pmatrix}\frac{\delta f}{\delta x}(x,y) & \frac{\delta f}{\delta y}(x,y)\end{pmatrix}$$

$$Dg(x,y)=\begin{pmatrix}\frac{\delta g_1}{\delta x}(x,y) & \frac{\delta g_1}{\delta y}(x,y)\\ \frac{\delta g_2}{\delta x}(x,y) & \frac{\delta g_2}{\delta y}(x,y)\end{pmatrix}\xrightarrow[y=0]{x=0}\begin{pmatrix}0 & 1\\ 3 & 4\end{pmatrix}$$

$$Df\circ g(0,0)=\begin{pmatrix}\frac{\delta f}{\delta x}(0,1) & \frac{\delta f}{\delta y}(0,1)\end{pmatrix}\begin{pmatrix}0 & 1\\ 3 & 4\end{pmatrix}=\begin{pmatrix}2 & 3\end{pmatrix}$$

$$3\frac{\delta f}{\delta y}(0,1)=2$$

$$\frac{\delta f}{\delta y}(0,1)=\frac{2}{3}$$

$$\frac{\delta f}{\delta x}(0,1)+4\frac{\delta f}{\delta y}(0,1)=3$$

$$\frac{\delta f}{\delta x}(0,1)+\frac{8}{3}=3$$

$$\frac{\delta f}{\delta x}(0,1)=\frac{1}{3}$$

$$z=f(0,1)+\frac{x}{3}+\frac{8}{3}(y-1)$$

$f(0,1)=f\circ g(0,0)$ Coincide con el plano tangente

$$f\circ g(0,0)=3+0+0=3$$

$$z=3+\frac{x}{3}+\frac{8}{3}(y-1)=\frac{1}{3}+\frac{x}{3}+\frac{8}{3}y=\frac{1+x+8y}{3}$$