

1)

$$G_n = 0 \forall n > 1$$

$$G_p^* = \{G_p\} - \{1\}$$

$$\sum G_p^* = \sum G_p - 1 = -1$$

$$\sum G_{pq}^* = \sum G_{pq} - \sum G_p^* - \sum G_q^* - 1 = 0 + 1 + 1 - 1 = 1$$

3)

$$n^n \equiv ?(5)$$

$$5 | n$$

$$n^n \equiv 0(5)$$

$$5 \nmid n$$

$$n^{4k} \equiv 1^k \equiv 1(5)$$

$$n^n \equiv n^{4k} n^{r_4(n)} \equiv n^{r_4(n)}(5)$$

$$\begin{array}{cccc} r_4(n) & 0 & 1 & 2 & 3 \\ r_5(n^{r_4(n)}) & 1 & r_5(n) & T1 & T2 \end{array}$$

T1:

$$r_5(n) \quad 1 \quad 2 \quad 3 \quad 4$$

$$r_5(n^2) \quad 1 \quad 4 \quad 4 \quad 1$$

T2:

$$r_5(n) \quad 1 \quad 2 \quad 3 \quad 4$$

$$r_5(n^3) \quad 1 \quad 3 \quad 2 \quad 4$$

$$r_{20}(n) \quad 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \quad 8 \quad 9 \quad 10 \quad 11 \quad 12 \quad 13 \quad 14 \quad 15 \quad 16 \quad 17 \quad 18 \quad 19$$

$$r_5(n) \quad 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4$$

$$r_4(n) \quad 0 \quad 1 \quad 2 \quad 3 \quad 0 \quad 1 \quad 2 \quad 3 \quad 0 \quad 1 \quad 2 \quad 3 \quad 0 \quad 1 \quad 2 \quad 3 \quad 0 \quad 1 \quad 2 \quad 3$$

$$r_5(n^n) \quad 0 \quad 1 \quad 4 \quad 2 \quad 1 \quad 0 \quad 1 \quad 3 \quad 1 \quad 4 \quad 0 \quad 1 \quad 1 \quad 3 \quad 1 \quad 0 \quad 1 \quad 2 \quad 4 \quad 4$$

$$r_5(n^n) = 0 \leftrightarrow n = 0, 5, 10, 15$$

$$r_5(n^n) = 1 \leftrightarrow n = 1, 4, 6, 8, 11, 12, 14, 16$$

$$r_5(n^n) = 2 \leftrightarrow n = 3, 17$$

$$r_5(n^n) = 3 \leftrightarrow n = 7, 13$$

$$r_5(n^n) = 4 \leftrightarrow n = 2, 9, 18, 19$$

	0	1	2	3	4
0	0	1	1	1	1
1	0	1	2	3	4
2	0	1	4	4	1
3	0	1	3	2	4

4)

(a)

$$aRb \leftrightarrow a = b$$

(b)

Imposible

(c)

$$aRb$$

$$bRa$$

$$aRa$$

$$bRb$$

(d)

$$aRb$$

$$bRa$$

$$bRb$$

(e)

$$aRb$$

$$bRc$$

$$aRc$$

(f)

$$aRb$$

$$bRc$$

(g)

$$aRb$$

$$bRa$$

$$bRc$$

$$aRc$$

(h)

$$aRb$$

$$bRa$$

$$bRc$$

5)

Hay solo 2 raíces sextas primitivas: G_1 y G_5

$$f(G_1) = 1 + G_5 - 3G_4 - 2 + G_2 - 3G_1 + a$$

$$f(G_1) = 0 \leftrightarrow a = 1$$

$$f(G_5) = 1 + G_1 - 3G_2 - 2 + G_4 - 3G_5 + a$$

$$f(G_1) = 0 \leftrightarrow a = 1$$

$$f(x) = x^6 + x^5 - 3x^4 + 2x^3 + x^2 - 3x + 1$$

$$= f_4(x) \left(x - \left(\frac{1}{2} + \frac{\sqrt{3}}{2}i \right) \right) \left(x - \left(\frac{1}{2} - \frac{\sqrt{3}}{2}i \right) \right)$$

$$= f_4(x)(x^2 - x + 1)$$

$$= (x^4 + 2x^3 - 2x^2 - 2x + 1)(x^2 - x + 1)$$

$$G: 1$$

$$= (x^3 + 3x^2 + x - 1)(x - 1)(x^2 - x + 1)$$

$$G: -1$$

$$= (x^2 + 2x - x)(x - 1)(x + 1)(x^2 - x + 1)$$

$$= -1 \pm \sqrt{2}$$