

11/2014

$$\sum_{i=4}^{2n} \bar{\omega}^{3i} = 0$$

$$\omega \in G_{15}^*$$

$$\sum_{i=0}^{2n} \bar{\omega}^{3i} - \sum_{i=0}^3 \bar{\omega}^{3i} = \frac{\bar{\omega}^{3 \cdot (2n+1)} - 1}{\bar{\omega}^3 - 1} - \frac{\bar{\omega}^{3 \cdot 4} - 1}{\bar{\omega}^3 - 1} =$$

$$= \bar{\omega}^{-6n+3}$$

$$\omega^{15} = 1$$

$$= \frac{\bar{\omega}^{-6n+3} + 1}{\bar{\omega}^{-3} - 1} - \frac{\bar{\omega}^{-12} + 1}{\bar{\omega}^{-3} - 1} = 0$$

$$\bar{\omega}^3 \neq 1$$

$$\Leftrightarrow \bar{\omega}^{-6n+3} = \bar{\omega}^{-12}$$

$$\text{div. por } \bar{\omega}^{-12} \neq 0$$

$$\bar{\omega}^{-6n+3-12} = 1$$

$$\omega^{-1} = \frac{\bar{\omega}}{|\omega|^2}$$

$$\Leftrightarrow \bar{\omega}^{-6n-9} = 1$$

$$\bar{\omega} = \omega^{-1}$$

$$\Leftrightarrow \bar{\omega}^{-6n+9} = 1$$

$$\Leftrightarrow 15 \mid -6n+9$$

$$\Leftrightarrow -6n \equiv -9 \pmod{15}$$

div. 3

$$\Leftrightarrow -2n \equiv -3 \pmod{15}$$

$$\Leftrightarrow \overbrace{7 \cdot (-2)n}^{\equiv 1 \pmod{15}} \equiv \overbrace{7 \cdot (-3)}^{-21} \pmod{15}$$

$$n \equiv 9 \pmod{15} \quad] \text{ Falta chequer!}$$