2)

a) $$\lim_{(xy)\to(1,0)} \frac{x^2 y \cdot sen\left(\frac{1}{y}\right)}{x^2+y^2} = \frac{1^2 0 \cdot aco}{1^2} = 0 \Rightarrow \text{Es continua}$$

$$\lim_{(xy)\to(0,0)} \left| \frac{x^2 y \cdot sen\left(\frac{1}{y}\right)}{x^2+y^2} \right| = \lim_{(xy)\to(0,0)} \frac{x^2 |y| \cdot \left| sen\left(\frac{1}{y}\right) \right|}{\|x,y\|^2}$$

b) $$\left| sen\left(\frac{1}{y}\right) \right| \le 1 \forall y$$

$$\lim_{(xy)\to(0,0)} \frac{x^2 |y| \cdot \left| sen\left(\frac{1}{y}\right) \right|}{\|x,y\|^2} \le \frac{x^2|y|}{\|x,y\|^2} \le \frac{\|x,y\|^2 \|x,y\|}{\|x,y\|^2} \le \|x,y\| = 0$$

$\Rightarrow$ Es continua

4)

a)
$$\lim_{xy\to(0,0)} \frac{f(xy) - f(0,0)}{\|(xy)\|} \stackrel{?}{=} 0$$

$$\lim_{xy\to(0,0)} \left| \frac{(x+y)\left(e^{xy}-1\right)}{\left(x^2+y^2\right)\|(xy)\|} \right| = \lim_{xy\to(0,0)} \left| \frac{(x+y)\left(e^{xy}-1\right)xy}{\|(xy)\|^3 xy} \right| = \lim_{xy\to(0,0)} \frac{|x+y||x||y|}{\|(xy)\|^3} < \lim_{xy\to(0,0)} \frac{2\|(xy)\|\|(xy)\|\|(xy)\|}{\|(xy)\|^3} < 2$$

Acercándome por rectas

$$\lim_{x0\to(0,0)} \frac{(x+0)\left(e^{x0}-1\right)}{\left(x^2+0^2\right)\|(x0)\|} = \lim_{x0\to(0,0)} \frac{0}{|x|^3} = 0$$

$$\lim_{xx\to(0,0)} \frac{(x+x)\left(e^{xx}-1\right)}{\left(x^2+x^2\right)\|(xx)\|} = \lim_{xx\to(0,0)} \frac{2x\left(e^{x^2}-1\right)}{2x^2\sqrt{2}|x|} = \lim_{xx\to(0,0)} \frac{x}{\sqrt{2}|x|} = \pm\frac{1}{\sqrt{2}} \ne 0 \to \nexists lím$$

No es diferenciable

b)
$$\frac{\delta f}{\delta v} = \lim_{h\to 0} \frac{f\left((x_0 y_0) + h(v_1,v_2)\right) - f(0,0)}{h\|v\|} = \lim_{h\to 0} \frac{f(hv_1, hv_2)}{h}$$

$$= \lim_{h\to 0} \frac{(hv_1+hv_2)\left(e^{hv_1 hv_2}-1\right)}{\left(h^2 v_1{}^2 + h^2 v_2{}^2\right)} = \lim_{h\to 0} \frac{h(v_1+v_2)\left(e^{h^2 v_1 v_2}-1\right)}{h^2\|v_1,v_2\|^2} = \lim_{h\to 0} \frac{(v_1+v_2)\left(e^{h^2 v_1 v_2}-1\right)}{h} \frac{hv_1v_2}{hv_1v_2}$$

$$= \lim_{h\to 0} (v_1+v_2) h v_1 v_2 = 0$$

Pero como dividí por hv_1v_2, este límite podría no dar 0 si $v_1 = 0$ o $v_2 = 0$

Es decir que tengo que probar con $\begin{matrix} v=(1,0) \\ v=(0,1) \end{matrix}$

$$\frac{\delta f}{\delta v}=\lim_{h\to 0}\frac{f((0,0)+h(1,0))-f(0,0)}{h}=\lim_{h\to 0}\frac{f(h,0)}{h}=\frac{\delta f}{\delta x}=\lim_{h\to 0}\frac{(h+0)(e^{h0}-1)}{(h^2+0^2)}=\lim_{h\to 0}\frac{0}{h}=0$$

$$\frac{\delta f}{\delta v}=\lim_{h\to 0}\frac{f((0,0)+h(0,1))-f(0,0)}{h}=\lim_{h\to 0}\frac{f(0,h)}{h}=\frac{\delta f}{\delta y}=\lim_{h\to 0}\frac{(0+h)(e^{0h}-1)}{(0^2+h^2)}=\lim_{h\to 0}\frac{0}{h}=0$$

$$\Rightarrow \frac{\delta f}{\delta v}(0,0)=0\forall v/\|v\|=1$$

5)

$$h(x,y,z)=\left(ze^{xy},y^2-sen(x^3z)\right)$$

$$g=f\circ h$$

$$Dg(0,1,3)=Df(h(0,1,3))\cdot Dh(0,1,3)$$

$$h(0,1,3)=(3,1)$$

$$Dg(0,1,3)=Df(3,1)\cdot Dh(0,1,3)$$

$$-x+2y-z=1$$

$$-x+2y-1=z$$

$$z=f(3,1)+\frac{\delta f}{\delta x}(3,1)(x-3)+\frac{\delta f}{\delta y}(3,1)(y-1)$$

$f(3,1)$ coincide con el plano tangente

$$f(3,1)=-3+2-1=-2$$

$$z=-2+x\left(\frac{\delta f}{\delta x}(3,1)\right)-3\left(\frac{\delta f}{\delta x}(3,1)\right)+y\left(\frac{\delta f}{\delta y}(3,1)\right)-\frac{\delta f}{\delta y}(3,1)$$

$$z=\left(-2-3\left(\frac{\delta f}{\delta x}(3,1)\right)-\frac{\delta f}{\delta y}(3,1)\right)+x\left(\frac{\delta f}{\delta x}(3,1)\right)+y\left(\frac{\delta f}{\delta y}(3,1)\right)$$

Como $\left(-2-3\left(\frac{\delta f}{\delta x}(3,1)\right)-\frac{\delta f}{\delta y}(3,1)\right)$ es una constante, lo que multiplica a *x* y a *y* en la ecuación del plano tangente es lo mismo.

$$Df(3,1)=\left(\frac{\delta f}{\delta x}(3,1),\frac{\delta f}{\delta y}(3,1)\right)=(-1,2)$$

$$Dh(0,1,3)=\begin{pmatrix} \frac{\delta h_1}{\delta x}(0,1,3) & \frac{\delta h_1}{\delta y}(0,1,3) & \frac{\delta h_1}{\delta z}(0,1,3) \\ \frac{\delta h_2}{\delta x}(0,1,3) & \frac{\delta h_2}{\delta y}(0,1,3) & \frac{\delta h_2}{\delta z}(0,1,3) \end{pmatrix}$$

$$h_1(x,y,z)=ze^{xy}$$

$$\frac{\delta h_1}{\delta x}=yze^{xy}$$

$$\frac{\delta h_1}{\delta x}(0,1,3)=3$$

$$\frac{\delta h_1}{\delta y}=xze^{xy}$$

$$\frac{\delta h_1}{\delta y}(0,1,3)=0$$

$$\frac{\delta h_1}{\delta z}=e^{xy}$$

$$\frac{\delta h_1}{\delta z}(0,1,3)=1$$

$$h_2(x,y,z)=y^2-sen\left(x^3z\right)$$

$$\frac{\delta h_2}{\delta x}=-\cos\left(x^3z\right)\left(3x^2z\right)$$

$$\frac{\delta h_2}{\delta x}(0,1,3)=0$$

$$\frac{\delta h_2}{\delta y}=2y$$

$$\frac{\delta h_2}{\delta y}(0,1,3)=2$$

$$\frac{\delta h_3}{\delta z}=-\cos\left(x^3z\right)\left(x^3\right)$$

$$\frac{\delta h_3}{\delta z}(0,1,3)=0$$

$$Dh(0,1,3)=\begin{pmatrix} 3 & 0 & 1 \\ 0 & 2 & 0 \end{pmatrix}$$

$$Dg(0,1,3)=\begin{pmatrix} -1 & 2 \end{pmatrix}\cdot\begin{pmatrix} 3 & 0 & 1 \\ 0 & 2 & 0 \end{pmatrix}=\begin{pmatrix} -3 & 4 & -1 \end{pmatrix}$$

$$\left\|\left(\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}},\frac{-1}{\sqrt{3}}\right)\right\|=1$$

$$\frac{\delta g}{\delta v}(0,1,3)=\begin{pmatrix} -3 & 4 & -1 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \\ 3 \end{pmatrix}=1$$